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On the Agrarian Measures of the Egyptians.

WHAT is at present practised in Egypt is a faithful representation of what has been practised from the earliest ages of civilization. Hence, the present practices will give us an idea of the knowledge possessed by the priests of that country. It is obvious, that in the measurement of lands much time would have been lost if they had measured the *aroura;* (a square whose side was 100 Egyptian cubits in length,) by applying successively a cubit measure along the line to be measured. They replaced the cubit by one of its multiples. The land measurer, holding in his hand a long reed, places himself at the extremity of the line to be measured; he traces with this reed a slight transverse furrow, to point out the place of that extremity; he places one end of the reed as near the ground as possible, and traces with the other end a second furrow; he places the end of his cane upon this second furrow, and thus he proceeds to measure the whole line. This mode of measuring is simple, is ready and easy in practice, but it is not exact. Because the unit of agrarian measures was a square whose side was 100 cubits, the length of the cane employed must have been a factor of this number. A reed of 5 cubits satisfies the conditions, and the unit of agrarian measure of 10,000 cubits was thus transformed into another of 400 square canes. To render the ope-

rations more expeditious was to solve a problem of great importance. The priests contrived a new cane. On constructing upon the diagonal of a square, a new square, we see that by prolonging the sides of the primitive square, we have the diagonals of the second, and that the second was exactly double the first. We perceive that the diagonal contains more than 28 canes, and less than 29; more than 141 cubits, and less than 142. They pitched upon 28 canes; the error was 16 superficial canes in 800, or a 50th part, and this was favourable to the government, because it increased the imposts. The number 28 has 7 for a divisor; on that account the cane was made 7 cubits long, still with the view of abridging. It is true that we do not find in antiquity any positive evidence of the employment of a cane of 7 cubits, but we can fortify our supposition by circumstances nearly as strong as if the fact had been mentioned. It is natural to suppose that the base of the great pyramid ought to contain a round number of lineal measures. It is natural to think that the base of this pyramid ought to contain a round number of superficial measures. According to the last measurement, the surface of the base is 54.135 metres, which makes exactly ten of these septennary arouras, and gives for the cubit 0.525 metres, exactly what is deduced from the sepulchral chamber, and likewise from the nilometer at Elephantine. We may admit the exactness of these coincidences, but adopting the whole hypothesis, it only follows that the Egyptian priests understood the most simple case of the famous problem of the square of the hypothenuse which does not indicate a very advanced state of the sciences. Respecting the agrarian measures in Egypt under the Persians and Romans, we see that the *jugerum* of Hero is nothing else but the Roman ugerum. We find it proved by a passage of Didymus, of Alexandria, that the Italian foot was the same as the Roman foot. All the modifications introduced into the agrarian measures, are explained by this principle, which has always regulated the

conduct of conquerors, to augment the sum of the impositions attending as much as possible to the habits of the conquered people. From calculation, it appears that the real size of the base of the great pyramid, is only the *one hundred and twenty-third* part different from the value asssigned to it by Pliny.

An improvement on the common Ship-Pump. *By* R. Patterson. (*From the Transactions of the American Philosophical Society. Vol.* 1. *New Series.*—1818.)

Notwithstanding the numerous improvements that have, from time to time, been proposed in the construction of ship pumps, yet, after all, the common *lifting pump* still remains in general use. This paper is an attempt towards an improvement in this pump, by means of a very simple appendage, that may be readily applied at any time when wanted, and by which a considerable proportion of the manual labour usually employed will be saved. The following is a description of this appendage, with the manner of its application and use. I. Let a plug of white pine, cedar, or any other suitable wood, or of cork, be made very nearly cylindrical, so as exactly to fit the bore of the pump above the nozle. II. Through the axis of this plug, a hole is to be bored of the size of the pump-rod, and then the plug is to be split, or cut through the axis, or centre of the hole. III. Place this plug round the pump-rod, and let it be firmly inserted in the bore of the pump, above the nozle, and there secured by a pin or bolt driven through the pump just above the plug, so as to prevent it from being raised by the force of the water acting against it. The part of the pump-rod that works in the plug may be made as round and smooth as possible, in order to prevent friction, and the passage of the water through the hole. With the same view, the hole may be lined or packed with oakum, and a stra-

tum of oil, or slush, placed over the plug. It will be advisable to have the nozle inserted as *low down* in the pump tree as practicable, and thus the vertical hole through the plug will be the less affected by the angular motion of the pump-rod. IV. Round the nozle of the pump let there be fastened one end of a pretty wide open hose of leather, or painted canvas ; the other end passing over or through the side of the vessel, and hanging down into the water. The pump, with this simple appendage, may be considered as a syphon, having the shorter leg outside, and the longer leg inside of the vessel ; and the height to which the water will in effect have to be raised, by the act of pumping, will be no more than the *difference* between the height of the water in the hold and that outside of the vessel ; and thus, frequently, more than half the labour of pumping will be saved.

Remarks.—I. The height to which the water will, *in effect*, have to be raised by pumping, is that stated above, on the supposition of the vessel being at rest, or in still water : But if under way, and sailing with any considerable velocity through the water, as is generally the case when the pump is most employed, then the labour of pumping will be still further diminished. For, it is easy to demonstrate, from the principles of hydraulics, that the velocity of the open end of the hose through the water will have the effect of raising the water from the hold to a height equal to that from which a heavy body descending would acquire that velocity ; neglecting the effects of friction. Thus a velocity of a little more than $4\frac{1}{2}$ knots per hour, would raise the water *one* foot, and a velocity of somewhat less than $9\frac{1}{2}$ knots would raise the water four feet, &c. &c. And this effect will be still further increased by the traction of the external water on that issuing from the hose ; which, in ordinary cases, will be far from inconsiderable. II. The best way of applying the labour of men, or other animals, in the working of pumps or other machines, is, when practicable, that in which both the

weight of the animal, and the *strength* of its muscles are employed; and in which short intervals of rest, or of greater and less exertion, constantly succeed each other. The action of rowing a boat will serve as a very good example of this application of labour. III. It follows, from the above principle, that the manner of applying the labour of man in working a pump, in the common way, is the most injudicious that can be conceived. His body serves only as a fulcrum for his arms to work on, its weight contributing little or nothing to the effect produced; this depending almost entirely on the exertion of the strength of his arms, and that in a direction which is, in general, the most fatiguing, and least effective possible: not to mention the constant reiterated *checks* which must be given to the action of the pump-handle, in its alternate up-and-down motion. IV. The manner of working the pump which I would propose, as free from all the above inconveniences and imperfections, is the following: 1. Let the pump-brake, or handle, be in the form and position of a pretty heavy pendulum, and of such a length that its natural oscillations may be nearly the same as those given it by the action of pumping. 2. Into this pendent pump-brake, let there be firmly fixed a long cylindrical pin, to serve as the immediate handle, and at right angles to the plane of the pendulum's motion. 3. Let the men working the pump be seated on a bench of a suitable height, with their feet pressing against a cleat or footstool, fastened to the deck; and in this position they will produce all the advantages which can possibly be derived from the exertion of animal force. 4. By placing the pendulum in such a position that its oscillations may correspond with the pitching or rocking of the vessel, the pumping in ordinary cases may, by this means, be frequently effected without manual labour. 5. By suffering the quantity of air to pass into the pump along with the water, the labour of pumping will be considerably alleviated, from the compressibility and diminished weight of the column of the

mixture of air and water in the pump-barrel ; and yet the total effect, in proportion to the quantity of force applied, will remain the same.

Description and rationale of the operation of a simple apparatus, which may serve as a substitute for the ship pump, and which will require no manual labour whatever. By R. PATTERSON.

Description.—The apparatus for the purpose announced in the above title, consists of a long hose, made of pretty stiff leather, passing through the stern of the vessel, the inner end furnished with a copper ferrule, and having a valve opening inwards, is to be immersed under the surface of the water in the hold, and the outer end to fall into the water a-stern of the vessel. This end of the hose is to terminate in a piece of copper tube, of a convenient length, with 3 or more large holes pierced through its circumference, near the extremity ; and to be closed at the end by a moveable lid, projecting a small distance beyond the circumference of the tube. This tube is to be introduced (the lid being removed for the purpose) into a broad metalic socket (bell metal or copper) from which project three or more diverging spiral tubes, opening into the socket ; which must be made to turn freely, and with as little friction as possible, round the copper tube, and covering the holes perforated through it ; the lid being replaced, will prevent the socket from slipping off. Round the socket, and behind the projecting spiral tubes are to be firmly fixed, obliquely, three or more copper vanes, resembling those of a vertical wind mill. Along the surface of the copper tube, in which the hose terminates, may be fixed an oblong sheet of cork, projecting a small distance above the tube. This will answer two purposes : 1. By its buoyancy, it will, when the vessel is in motion, prevent the

spiral tubes from sinking too much below the surface of the water; and, 2. It will counteract the tendency which the friction of the socket turned round by the rotary motion of the vanes striking against the water, will have to twist the leather hose. That part of the hose which, passing through, comes in contact with the stern of the vessel, may be made of a strong curving copper tube, by which it may be fastened to the vessel, and thus be prevented from being dragged out, or twisted round by the action of the water. Into the upper bend of this part of the hose may be inserted a small diverging copper tube, through which, by means of a funnel, the hose may be filled with water, or the air which may there accumulate suffered to escape, and may then be stopped with a cork.

Rationale of the operation. The hose being previously filled with water, and the vessel under way, the action of the water against the vanes attached to the socket will, in ordinary circumstances, produce as great a centrifugal velocity in the outer extremities of the spiral tubes, as to overcome the external pressure of the water, and produce a current from the water in the hold, on the principles mentioned in the above paper, so long as it covers the inner extremity of the hose. If the motion of the vessel should cease, or become too slow to produce the exhaustion of the water from the hold, then the valve at the inner extremity of the hose will be shut, and the hose remain full, till a favourable change of circumstances shall renew the operation. There is no doubt, that the above apparatus is susceptible of various modifications and improvements, which will readily occur to the practical navigator. A centrifugal pump is not a new idea. I remember to have seen one in Buck's county above fifty years ago, constructed by Joseph Ellicot, the father of our associate Andrew Ellicot, by which water was raised from a pretty deep well, for the purpose of irrigation, the rotary motion being communicated to the pump by a simple wind-mill.

Extract from No. IX. of the Transactions of the American Philosophical Society, vol. 1 New Series, being "An attempt to ascertain the fusing Temperature of Metals. By JOSEPH CLOUD.

The dilation observable in the fusion of metals, is a proof that the particles are separated, and kept at a distance from each other by the interposition of caloric between their integrants, sufficient to overcome their attraction of cohesion and their *inertiæ*. And when certain degrees of temperature are excited, they lose their solidity, and become fluid. From these general laws of fusion, it necessarily follows, that the melting heat of a metal will be governed both by its attraction of cohesion, and inertiæ, and that the comparative fusibility of the metals will be in the compound ratio of their attraction of cohesion and specific gravity.* In order to illustrate this position, I have availed myself of Mr. Gayton Morveau's experiments on the attraction of cohesion of the metals, by which he found that wires of 0.787 of a line in diameter required the following forces to tear them asunder: Iron, 549.250 lbs.; copper, 302.278 lbs.; platinum, 274.320 lbs.; silver, 187.137 lbs.; gold, 150.753 lbs.; zinc, 109.540 lbs.; tin, 34.630 lbs.; lead, 27.621 lbs. I shall make use of the specific gravities as stated by chemical authors. Iron, 7.788; copper, 8.667; platinum, 23.543; silver, 10.510; gold, 19.361; zinc, 6.861; tin, 7.299; lead, 11.352. Now, it has been ascertained that, in the fusion of tin, 442 degrees of Fahrenheit's scale are required to overcome the combined powers of 34.630 attraction of cohesion, and 7.299 of inertia (spec. grav.) I have taken them as a standard to find the melting temperature of the other metals, by the following pro-

* As the tendency in bodies to be at rest, and, consequently, the force required to put them in motion, depends upon their weight, their specific gravity furnishes us with an easy and correct method of ascertaining their comparative inertiæ.

portion: If 34.630 multiplied by 7.299 require 442 degrees, what will 150.735 multiplied by 19.361 (the attraction of cohesion and spec. grav. of gold) require. The answer will be 5.103 degrees of Fahrenheit, the fusing point of gold. By proceeding in the same manner with the other metals, we obtain the following results as their fusing temperature. Platinum 11.293° Iron 7.480°, Copper 4.581°, Silver 3.439°, Zinc 1.314, Lead 548°, 3. This last turns out to be the precise temperature at which Sir I. Newton found lead to melt.

Kidney Bean and Common Bean Perennials.

It is a generally received opinion, supported by botanical and horticultural writers, that the *phaseolus vulgaris*, or common kidney bean, and the *phaseolus manus*, or dwarf kidney bean, are annual plants. Experience, however, has taught me that they are both perennials, as well as the *faba vicia*, or common garden bean, with its varieties. The fact is readily proved. In the month of September, or October, on the appearance of frost, let them be cut off within two inches of the ground; shake over the roots some litter from a stable, and about the May following the roots will throw up fresh shoots, which will be stronger and more vigorous than those of the preceding year. This I have repeated with success for six years, in which time I have observed that the bean pods do not come to maturity so early by about three weeks, in the second, or succeeding years, as they do the first year's growth. But the second year's crop is not so liable to be injured by the weather, as to frosts, rain, &c. as the fresh sown plants.

On Malt.

According to M. Proust, the constituents of malt are Resin, 1; Gum, 15; Sugar, 15; Gluten, 15; Starch, 56; Hordein, 12; total 100. He affirms that barley in malting loses one third of its weight; but this is not accurate. The average loss, on more than 50 malting processes, on a large scale, which I superintended, and in which the greatest care was taken to ensure accuracy, was only 20 per cent. The malt was weighed just when taken from the kiln, and the barley just before it was put to steep. I found that if the barley was kiln dried, it lost 12 per cent. of its weight, and that the malt, when kept for some time in the granary, recovered the same proportion of weight; hence I conceive that the true loss in weight does not exceed 8 per cent. One half of this loss is owing to matter dissolved from the husk of the grain while in steep, and to grains bruised and destroyed by the maltster while turning it on the floor; so that the real loss does not exceed 4 per cent.

Reply to Pliny's letter to Lucinius.

The most probable cause of this curious phenomenon, is, that some pipe or hollow in the earth, bent in the form of a syphon, communicates with the bottom of the fountain; and proceeding thence towards the Larian lake, opens at some place lower than the bottom of the fountain. Admitting this to be the case, when the water in the fountain has risen above the bend of the pipe, it will flow through into the lake, and will continue flowing until the fountain be emptied; it will then cease to flow,

until the fountain be again filled above the bend of the pipe, when it will again begin to flow, and so on alternately; whence the cause of the phenomenon is evident.

W. Allen.

SOLUTIONS TO PHILOSOPHICAL QUESTIONS.

Solution to the 7th Philosophical Question, by Mr. O. Shannessy, Albany.

It has been ascertained, by the writers on Fluxions, that the centre of gyration of a *solid* sphere is nearer to its centre than that of a *hollow* one of the same diameter and quantity of matter is to its centre. And, indeed, it is evident; because the inertia of each is as its quantity of matter multiplied by the square of its distance from the centre. Now, in the proposed gold and silver balls, the entire mass of the former is posited towards the surface, and its distance from the centre is greater than that of the latter. And as these masses are the same, it follows that the inertia of the gold sphere is greater than that of the silver one; whence the following artifice. Let the two spheres roll down an inclined plane, observe that which rolls the quickest, or which has the greatest momentum in meeting the horizontal plane, for that is the ball of gold.

Mr. Gillman also sent an answer.

Qu. 8, *Answered by J. B——n.*

These stones generally appear luminous in their descent, moving in oblique directions, with very great velocity, and commonly with a hissing noise; they are frequently heard to explode or burst; they often strike the earth with such force as to sink several inches below the surface. They are

always different from the surrounding bodies, but in every case similar to each other, being semi-metallic, coated with a thin black incrustation. They bear strong marks of recent fusion. Chemists have found, that on examining these stones, they very nearly agree in their nature and composition, and in the proportions of their component parts. From this it is reasonable to conclude that they have all the same origin. To account for this phenomenon various hypotheses have been proposed. I shall just mention three. 1. That they are little planets, which, circulating in space, fall into the atmosphere, which by its friction diminishes the velocity, so that they fall by their weight. 2. That they are concretions formed in the atmosphere. 3. That they are projected from lunar volcanoes. These are the most probable conjectures ; and of these the two first possess a very small share of probability; but there are very strong reasons in favour of the last. As, 1. Volcanoes have been observed in the moon by means of telescopes. 2. The lunar volcanoes are very high, and the surface of the globe suffers frequent changes, as appears by the observations of Schroëter. 3. If a body be projected from the moon to a distance greater than that of the point of equilibrium between the attraction of the sun and moon, it will, on the known principles of gravitation, fall to the earth. 4. That a body may be projected from the lunar volcanoes beyond the moon's influence, is not only possible, but very probable : for, on calculation, it is found, that four times the force usually given to a 12 pounder will be quite sufficient for this purpose. It is to be observed, that the point of equilibrium is much nearest the moon, and that a projectile from the moon would not be so much retarded, as one from the earth, both on account of the moon's rarer atmosphere, and its less attractive force.

Mr. Copsey. These stones, on being analyzed, are found to be of a feruginous nature, and contain a great deal of lime and

sulphuric acid. Perhaps the formation of meteorolites may be accounted for on this principle, by supposing that when the electric fluid rushes from one cloud to another, or from a cloud to the earth, it carries with it all the various particles which have been raised into the air with the vapours by the heat of the sun, and which being thus forcibly brought into contact, become a solid body, which by its weight descends immediately to the earth. *Mr. Harrison.* Sir H. Davy has rendered it extremely probable that the newly-discovered metals, silicum, alumium, calcium, and magnium, are capable of being dissolved in hydrogen gas, and on this account I think it easy to give a concise explanation of the cause of the fall of meteorolites. We know that hydrogen gas is capable of dissolving arsenic, sulphur, phosphorus, and carbon, and of forming, respectively, arseniated, sulphurated, and phosphorated hydrogen; and meteorolites generally consist of silica, alumina, and magnesia, and the metals chromium, iron, manganese, and some others. Since, then, Mr. Davy has proved that hydrogen is capable of dissolving several of the metals, why, from analogy, may not hydrogen be capable, under favourable circumstances, of dissolving iron, manganese, &c.? Then, if a mixture of the solutions of a metal in hydrogen be ignited by an electric spark, may not metallic particles be precipitated, and by the agency of some mechanical operation, compressed into a comparatively small bulk, and appear under the form of a solid? Mr. J. Smith, R. T. junior, Mr. Baines, and Mr. Bamford, favoured us with similar opinions.

Qu. 10, *answered by Y. of New-Haven.*

If the pellicle of fluid enclosing a bubble of air be supposed to be acted on merely by the cohesive attraction of its parts, it will assume a spherical form, because, the solid content being given, the superficies will then be the least possible, and consequently, the cohesive attraction most satisfied. But if a sphe-

rical bubble be supposed to rest on the surface of a fluid of the same kind, the lower part of the film surrounding it will, by the united forces of gravitation and the cohesion of the adjacent surface of the fluid, be incorporated with that surface. Hence, the bubble (independently of the attraction of gravitation which may be considered as not sensibly affecting the figure) will assume that form, which, on a given base, and with a given solidity, has the least convex superficies possible; and this, it is well known, is the segment of a sphere. This is on the supposition that the gravitation of the particles composing the pellicle produces no sensible effect in comparison with their cohesion. So far as the influence of gravitation is admitted, the superficies will deviate from the spherical form, and approach that of the solid, generated by the revolution of a catenary. If the density of the inclosed air be supposed less than that of the atmosphere, its form will be still farther modified.

Qu. A. *answered by Y. of New-Haven.*

If the arc of a great circle be not the shortest distance between any two points on the surface of a sphere, let the line be drawn which is the shortest; let it be denoted by B, and the arc of the great circle by A. With one of the given points as a pole, let parallel small circles be drawn intersecting these two lines, and through each of the intersections of these parallels with B. draw perpendicular arcs of great circles. A series of triangles will then be formed, except where B is parallel to A, of which the portions of B will be hypotheneuse, and the corresponding portions of the parallels and great circles last drawn will be bases and perpendiculars, respectively. These triangles may be so increased in number, and diminished in magnitude, as to approach nearer than by any given difference, to right angled plane triangles. Now, because B does not coincide with A, a definite part of the power must be oblique to the latter. And since the hypotheneuse is greater than the perpendicular of a right angled triangle, the sum of the hypothenuses

which make up this oblique portion will be greater by a finite difference than the sum of the perpendiculars, which is evidently equal to the corresponding portion of A. The remaining portions are not *less* than the corresponding parts of A; hence A is shorter, by a finite difference, than any other line B, which can be drawn between the same given points.

Otherwise. Whatever be the shortest line between two points on a spherical surface, if the distance between these points be divided into two equal parts, the parts of the curve on each side of the point of division must each be a minimum. Hence, as a minimum is determined by exactly the same conditions in both, these parts must be equal and similar to each other. As the same reasoning holds true for any number of divisions whatever, it follows that the curvature of the shortest line is every where equal, and hence that it is a circular arc. But if it be an arc of a circle of the sphere, it must be an arc of a great circle; for, as is demonstrated in Legendre's Geometry, if it be not admitted as an axiom, (which is done by Playfair, Book I. Sup. Euc.) the arc of a less circle is greater than that of a larger on the same chord.

PHILOSOPHICAL QUESTIONS,

PROPOSED FOR DISCUSSION IN FUTURE NUMBERS.

Qu. 1. *By Mr. J. Laidlaw, Brooklyn.*

What is the reason that a human body sinks in water, after being drowned, but, in a few days, rises, and floats on the same fluid?

Qu. C. *By my uncle Toby.*

My uncle Toby has a conical piece of timber in one of the salient angles of his fortification—the altitude 3, and base diam-

eter 6 feet—Trim said it was bullet proof—my uncle Toby said no—I will try it with a ball of six inches in diameter, says Trim —do corporal, says my uncle Toby. Well, your honour was right, said Trim, returning—the centre of the ball passed through the middle of the cone, just one foot above the ground, and now sticks in the citadel. My uncle Toby smiled—Trim stared. I will tell thee what, Trim, said my uncle Toby—the corporal bowed—I will tell thee Trim, I want to have the wood that is left measured—and who shall we get to do it? said Trim, your honour knows I am not scholar good enough—Write it down, Trim, and send it to the Scientific Journal, and perhaps we shall get an answer to it—your honour shall be obeyed, said Trim; but what does your honour want to know—Why, Trim, say that I want to know the *solidity* and *superficies* of what remains of my cone—and, hark ye, Trim! tell the Editor that I will treat the gentleman that answers it with a bottle of sack, and that he may tell this to all his friends—They can't do it, said Trim—they *shall* do it, says my uncle Toby.

TO CORRESPONDENTS.

We beg of our ingenious correspondents to forward their favours before the 20th day of the month, as we are obliged to forward the matter about that time to the Printer. Questions requiring diagrams have been postponed, on account of the increased expense, but they shall be inserted as soon as we have subscribers enow to cover the expense of publishing, which we hope soon will be the case, as the numbers on the list are rapidly increasing. To question B, we have not yet received any solution.

MATHEMATICAL CORRESPONDENCE.

Qu. 11. *answered by Y. of New-Haven, and Zero.*

This question is somewhat indefinitely worded, I shall take it for granted that simple interest is intended, and that their shares in the profits, before the allowance is made to A, are supposed equal; the 15 per cent. on B's share is 375, which deducted from 1420 the amount of A's debt, leaves a balance of 1045 dollars in favour of B. Mr. *O'Connor.* The interest of 1000 dollars for 6 years at 7 per cent. is 420, which we suppose A must pay out of his share of the profits, and since he must have 15 per cent. for managing, &c. this on B's half would be 375. Hence A's account would stand thus $2500 + 375 - 420 = 2455$, and B's thus $2500 + 420 - 375 + 1000 = 3545$. This solution is on the supposition that A allowed B interest during 6 years, and also half the profits, and that at the end of that period they "squared off." *Mathetes* makes A's share 2875 dollars and B's 3125, which are as 3 to 5. Mr. *J. C. Strode, Burlington.* The amount of 1000 dollars in 6 years at 7 per cent. is 1420 = what A is indebted to B at the end of that time; now the allowance made by B to A for managing &c. $= 2500 \times ,15 = 375$ which subtracted from A's debt, gives 1045 dolls. in favour of B.

Qu. 12. *answered by Mr. O'Shannessy, Albany.*

As the horse moves with twice the celerity of the man, it is hence evident that the horseman overtakes the footman at the end of the 2. 4. 6. 8. and 10 miles; also, that the horse remains as long tied as he would be in uniformly performing half the journey, therefore 8 m. : 1 h. :: 15 m. : 1 h. $52\frac{1}{2}$ m. the time

sought. *Y. of New-Haven.* B walks his 5 miles in $\frac{5}{4}$ hours, and rides the remaining 5 in $\frac{5}{8}$ hours, therefore the journey is performed in $15 \div 8$ hours. B overtakes A at the end of every second mile, and the horse is at rest one third of the time. Mr. *J. Laidlaw.* The horse goes a mile in $\frac{1}{8}$ of an hour, and a man in $\frac{1}{4}$ of an hour; it therefore requires either of them $\frac{3}{8}$ hours to travel 2 miles, at which time they will both be together; consequently they must be so at the end of every two miles, and $5 \times \frac{3}{8} = 15 \div 8 = 1\frac{7}{8}$ hours is the time sought. *Zero's* answer is exactly in the same manner. Mr. *Nolan* solved it in the same manner as Y. Mr. *Strode.* In $\frac{3}{8}$ of an hour A and B will come together at the end of every 2 miles; hence $2 : 10 :: \frac{3}{8} : 1\frac{7}{8}$ hours, the time required.

Qu. 13. *answered by Mr. O. Reynolds, Professor, Baltimore.*

Put x = youth's age, then by the nature of the question $8x + 4$, and $3x^2 + 9$ must be squares, and their product $36 + 72x + 12x^2 + 24x^3$ = a square, assume $6 + ax + bx^2$ for its root, then we shall have $36 + 72x + 12x^2 + 24x^3 = 36 + 12ax + (12b + a^2)x^2 + 2abx^3 + b^2x^4$; and by comparing the coefficients of the like powers of x, we find $12a = 72$, and $12b + a^2 = 12$, hence $a = 6$, and $b = -2$; also making $2abx^3 + b^2x^4 + 24x^3$, gives $x = (24 - 2ab) \div b^2 = 48 \div 4 = 12$, the age required. Mr. *J. C. Strode.* It is evident that $8x + 4$ will be a square when $x = 2y$, and $y - 4 = 2$; whence $y = 6$, and $x = 2y = 12$. Again $3x^2 + 9$ will be a square when $x = 6v$, and $3v^2 - 9 = 3$; from which $v = 2$, and $x = 6v = 12$; therefore 12 will answer the conditions of both equations. *Mathetes.* The question requires us to make $8x + 4$, and $3x^2 + 9$ squares. Since $8x + 4$ = a square, $2x + 1$ = a square, which equate with $1 + 4y + 4y^2$, then $x = 2y + 2y^2$, and $3(2y + 2y)^2 + 9$, or $9 + 12y^2 + 24y^3 - 12y^4$ must be a square. Let $3 + 2y^2$ be its root, and we have $9 + 12y^2 + 24y^3 + 12y^4 = 9 + 12y^2 + 4y^4$, from whence y

$= - 3$, and $x = 12$ the age required. Mr. *O'Connor*. Put $8x + 4 = z^2$, then $x = (z^2 - 4) \div 8$ and $x^2 = (z^2 - 4)^2 \div 64$. Substitute this value of x^2 in the 2. given expression, and we have $\dfrac{3z^4 - 24z^2 + 48}{64} + 9 =$ a square, and multiplying by 64 = a square we have $3z^4 - 24z^2 + 48 =$ a square. In this expression 2 is the least integral value of z that will render it a square. Let $z = 2 + y$, then by substit. and reduction, $48y^2 + 24y^3 + 3y^4 + 576 =$ a square. Assume $y^2 + 24$ for its root, then $y^4 + 48y^2 + 576 = 48y^2 + 24y^3 + 3y^4 + 576$, hence $-2y^4 = 24y^3$, and $y = -12$, consequently $z = 2 + y = -10$, and $x = (z^2 - 4) \div 8 = 12$.

Qu. 14 *answered by Mr. Nolan, and Y. of New-Haven.*

Let x and y denote the contiguous arcs, and r the radius. If a denote the line joining the extremities of the sines, a and r will be diagonals of a quadrilateral, the sides of which are sin x, cos x, sin y, cos y, and since the rectangle of the diagonals of a quadrilateral, whose opposite angles are equal to two right angles, is equal to the sum of the rectangles of the opposite sides, $ra = \sin x \cos y + \cos x \sin y$. Dividing both sides by r, we obtain for the value of a the common expression for the sine of the sum of the arcs x and y. The answer by *Zero* is almost identical with the above. Mr. *O'Shannessy*, Let A, B, represent any two arcs, then (Eu. 22. 3. and data 6.) the straight line joining the intersections of those arcs with their limiting diameters is $= \sin A \cos B + \sin B \cos A$, which is well known to be $= \sin (A + B)$. The solution by *Mathetes* is almost in the same words. Mr. *O' Connor's* is very elegant, and purely geometrical.

Prize Qu. 15. *answered by Y. of New-Haven.*

Let the given distance of the centre of gyration from the centre of motion $= a$. From the nature of the centre of gyration, a

weight applied at any variable distance x from the centre, will produce the same angular velocity in the wheel as if the whole quantity of matter were collected uniformly in the circumference at the distance a from the centre. If the moving force be represented by W, and the weight of the wheel by w, the inertia of the latter will be equal to the quantity of matter $\frac{a^2 w}{x^2}$ collected in the circumference at the distance x. Hence the quantity of matter moved is properly denoted by $W + \frac{a^2 w}{x^2}$; and the accelerating force at the distance x will be as $\frac{W}{W + \frac{a^2 w}{x^2}}$ or as $\frac{W x^2}{W x^2 + w a^2}$. The distance described in a given time is as the accelerating force; for it is demonstrated by writers on mechanics, that the motion of revolving bodies acted on by weights, is uniformly accelerated; and the number of revolutions in a given time is as the distance described by the weight directly, and the distance from the centre at which it acts inversely; we have therefore to make $\frac{W x^2}{W x^2 + w a^2} \times \frac{1}{x} = \frac{W x}{W x^2 + w a^2}$ a maximum. If the differential of this expression be made $= 0$, by reduction x is found $= a \sqrt{} (w \div W)$; and when $a = 10$, $W = 10$, and $w = 40$, $x = 20$. Or since $a = r \div \sqrt{} 2$, the weight ought to be at $\sqrt{} 2$, or at 1.414214 times the radius of the wheel from the centre.

Again, by Zero.

Let w = weight of the wheel, r = distance of the centre of gyration from the axis of motion, m = weight applied to move

the wheel, and $x =$ the distance at which m is applied to produce the greatest angular motion; then will $\frac{w\,r^2}{x^2}$, be the quantity of matter which when placed at the distance x from the axis of motion, will have the same inertia as the wheel, therefore m is to move the matter $\frac{w\,r^2}{x^2}$. Now $m + \frac{w\,r^2}{x^2}$ is the matter moved, and m being the moving force, we have $m \div \frac{w\,r^2 + m\,x^2}{x^2}$, or $\frac{m\,x^2}{w\,r^2 + m\,x^2} =$ the accelerating force; this divided by x gives $\frac{m\,x}{w\,r^2 + m\,x^2}$, which varies with the angular motion, and by the question must be a maximum. This last expression put into fluxions, and equated to nothing; by reduction we obtain $x = \sqrt{w\,r^2} \div \sqrt{m} = \sqrt{400} = 20$; the distance required.

Another answer by Mr. O' Connor, New-York.

Put the given distance $= a$, the weight of the wheel $= 40 = b$, the moving power $10 = c$, and the required distance $= x$. Then by mechanics $x^2 : a^2 :: b : b\,a^2 \div x^2 =$ the weight to be substituted at the required point, and which being added to c gives $\frac{a^2\,b}{x^2} + c = \frac{a^2\,b + c\,x^2}{x^2}$ for the mass to be moved by the gravity of c, consequently, $\frac{a^2\,b + cx^2}{x^2} : c :: c : c^2\,x^2 \div (a^2\,b + cx^2) =$ the portion of its own weight by which the mass c is moved. Now since the motions of c, and of the centre of gyration must necessarily be coincident in point of time, the max. vel. of the latter must be coincident with the former; but the velocity of each is as its distance from the centre of motion, and

also as the force of gravity, therefore $x : a :: c^2 x^2 \div (a^2 b + cx^2)$ $: a c^2 x \div (a^2 b + cx^2) =$ the velocity of c, a max. By taking the fluxion of this expression, and reducing the result, we obtain $x^2 = a^2 b \div c$, and $x = \sqrt{ab} = \sqrt{(10 \times 40)} = 20$, the distance required.

Again, by Mr. Rt. Maar.

Put the given weight of the fly $= 40 = m$, the distance of the centre of gyration from the axis of motion $= 10 = r$, the moving power $= 10 = p$, and x the distance at which p must be applied to produce, in a given time, the greatest number of revolutions. First $mr^2 \div x^2 =$ the resistance of the wheel reduced to p, and $p + \dfrac{mr^2}{x^2} =$ mass moved, and the moving force p divided by this, gives $px^2 \div (mr^2 + px^2)$ for the accelerating force. But the number of revolutions is as the space the weight describes directly, and inversely as x, its distance from the centre of motion; therefore, the number of revolutions is as $px \div (mr^2 + px^2)$; which expression put into fluxions, and reduced, x comes out $= r \sqrt{\dfrac{m}{p}} = 10 \sqrt{\dfrac{40}{10}} = 20 =$ the distance at which the power ought to be applied.

The prize is awarded to *Y of New-Haven.*

Mathematical Questions, to be answered in No. 7.

Qu. 28. By *Gregory M'Gregor.*

Determine the segment of a sphere, such, that its *solidity* may have to its *curve surface*, the greatest ratio possible.

Qu. 28 By *Y. of New-Haven.*

In throwing n dice, what is the chance in favour of any given number (not greater than $6\,n$ nor less than n) being turned up?

Qu. 30. By *Mr. Turner.*

The sum of the squares of two numbers being 41, and the sum of their cubes = 189; to determine the numbers.

Qu. 31. By *Mr. O. Shannessy, Academy, Albany.*

Find two numbers whose sum shall be an integral cube, product a perfect square, and such that the square of the first plus the second, may be equal to the square of the second plus the first.

Qu. 32. By *Mr. C. Davis.*

Between 9 and 10 the hour and minute hands of a clock made an angle of 5° with each other; required the exact time?

Qu. 33. By *M. P. Cheyney, Burlington.*

Admit a man receives 6000 dollars, and out of this number reserves 400, and puts the remainder out at interest at 6 per cent. per ann. now, if he reserves 400 dollars every year until the whole is exhausted, how long will it last him, reckoning compound interest.

Qu. 34. By *Zero.*

Let there be two cylinders of equal diameters, their axes being in the same plane perpendicular to each other; supposing that one perforates the other, it is required to find the solidity and surface of the solid which shall so fit the hole made in the perforated cylinder as completely to fill it, in such a manner that the cylinder may appear entire.

List of names of the gentlemen who answered the questions for No. 4.

Analyticus, New-York, . .	11.	12.	13.	14.	15.
Mr. Rt. Maar, New-York, . .	11.	12.	13.	14.	15.
Mr. P. Nolan, New-York, .	11.	12.	13.	14.	15.
Mr. M. O'Connor, New-York, .	11.	12.	13.	14.	15.
Mr. O'Shannessy, Albany. .	11.	12.	13.	14.	15.
Mr. O. Reynolds, Baltimore, .	11.	12.	13.	14.	15.
Mr. J. C. Strode, Burlington, .	11.	12.	13.	14.	
Mathetes, near Philadelphia, .	11.	12.	13.	14.	
Mr. Tomlinson, . . .	11.	12.	13.	14.	15.
Tyro, Princeton, . . .		12.			
X + Y, Princeton, . . .			13.		
Y of New-Haven, . . .	11.	12.	13.	14.	15.
Zero, ...	11.	12.	13.	14.	15.
Mr. W. Wood, ...	11.	12.	13.		

Zero's solution to Qu. A, will be attended to in a future number.

On account of some alterations making in the printing-office, No. 4 did not come out till the 10th inst. we hope this will not again take place, and trust our correspondents will excuse the delay.